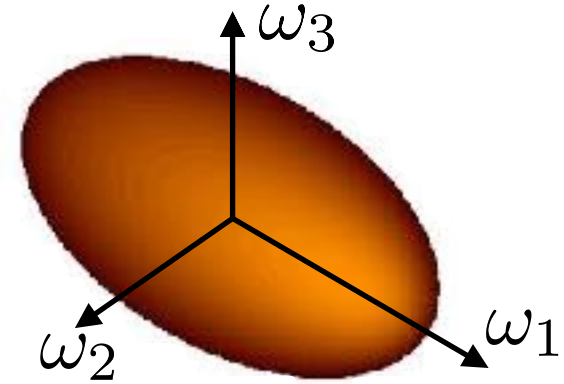
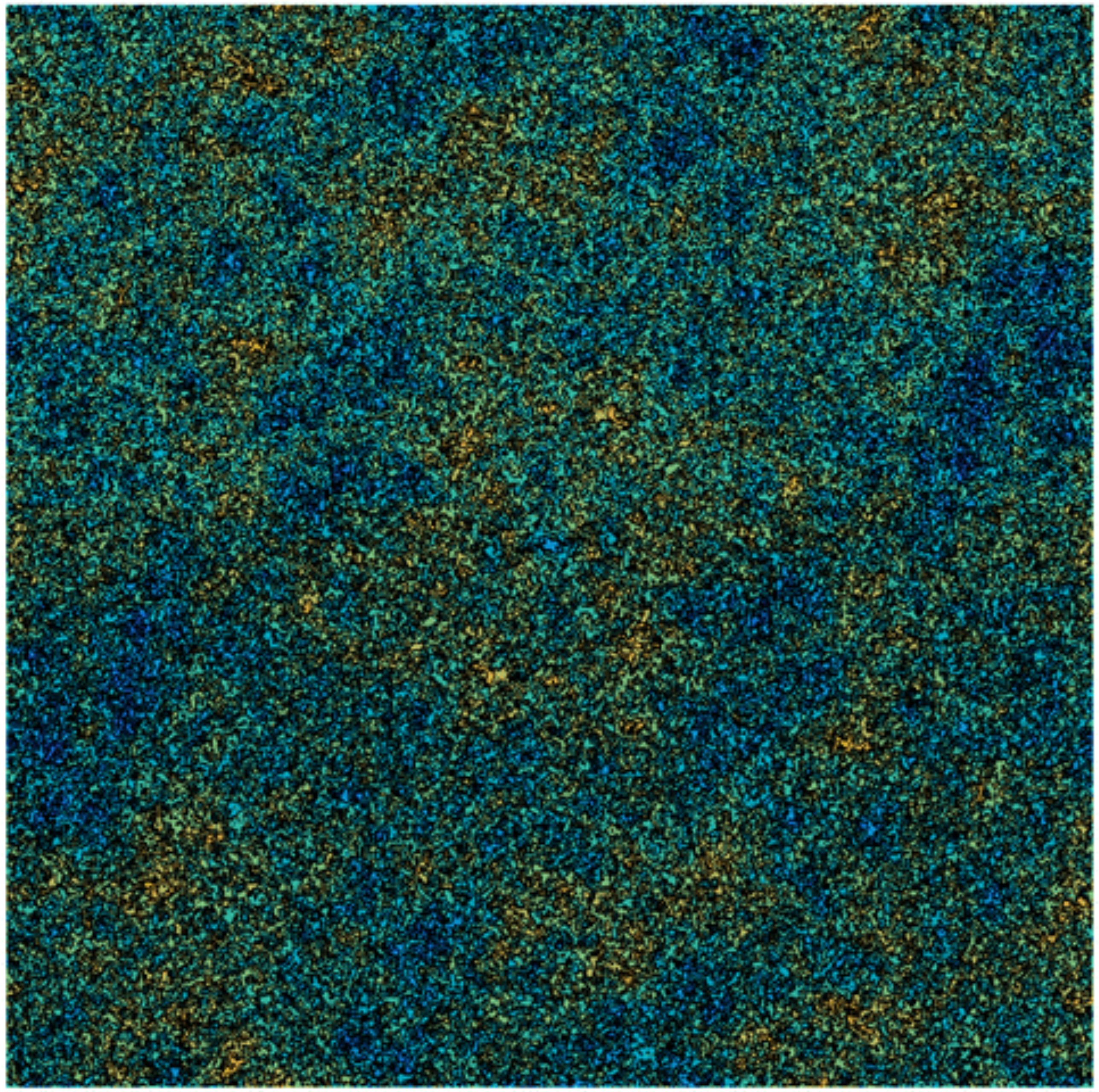
EDPs

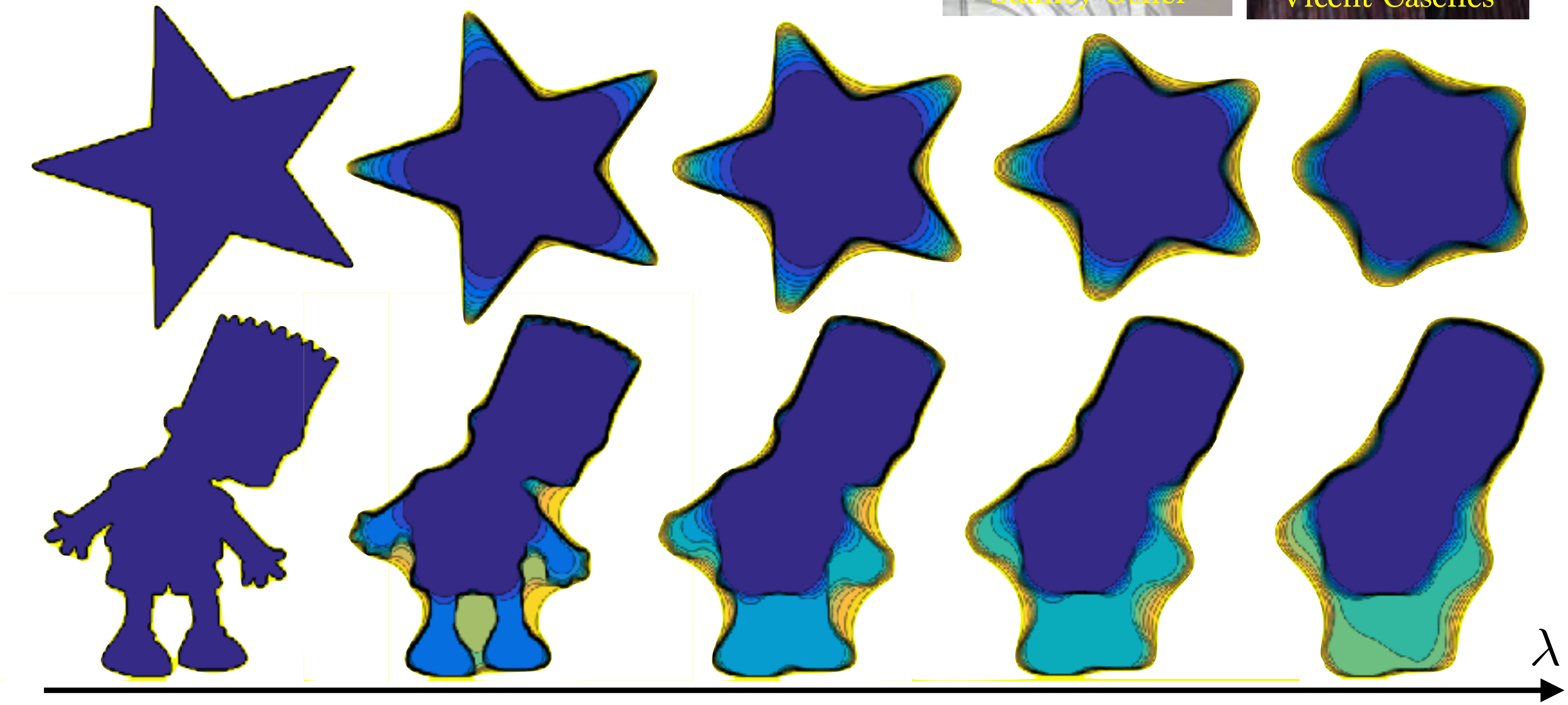
$$\|f\|_\alpha^2 = \sum_\omega \|\omega\|^{2\alpha} |\hat{f}(\omega)|^2 \leqslant 1$$





Rudin-Osher-Fatemi model:

$$\min_f \int |f(x) - y(x)|^2 + \lambda \|\nabla f(x)\| \mathrm{d}x$$



Parametrization (explicit)

$$\{\gamma(t)\ ;\ t \in [0,1]\}$$

Normal: $n = \dfrac{\gamma'}{\|\gamma'\|}$

Curvature: κ

ODE curve evolution

$$\frac{\mathrm{d}\gamma}{\mathrm{d}t} = \alpha(\gamma, n, \kappa) n$$

Levelset (implicit)

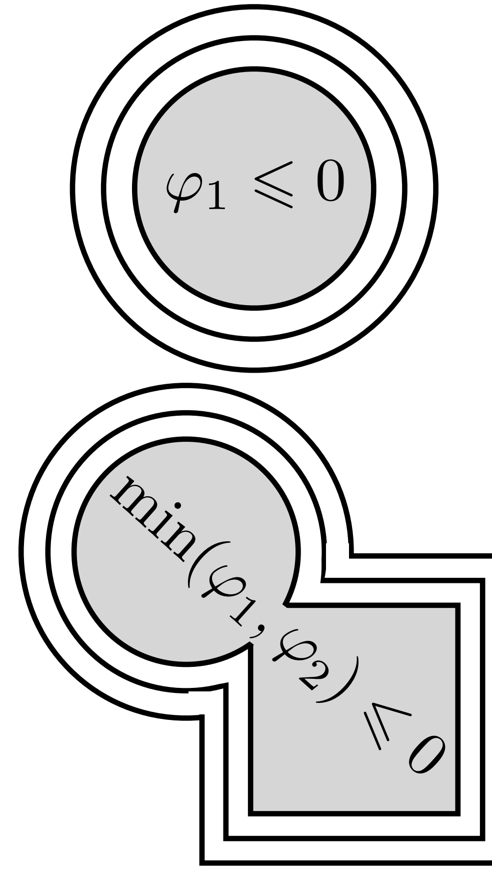
$$\{x\ ;\ \varphi(x) = 0\}$$

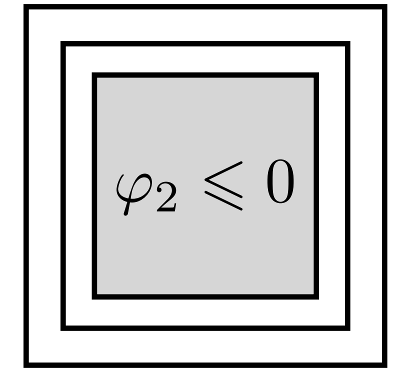
$$\frac{\nabla \varphi}{\|\nabla \varphi\|}$$

$$\mathrm{div}\left(\frac{\nabla \varphi}{\|\nabla \varphi\|}\right)$$

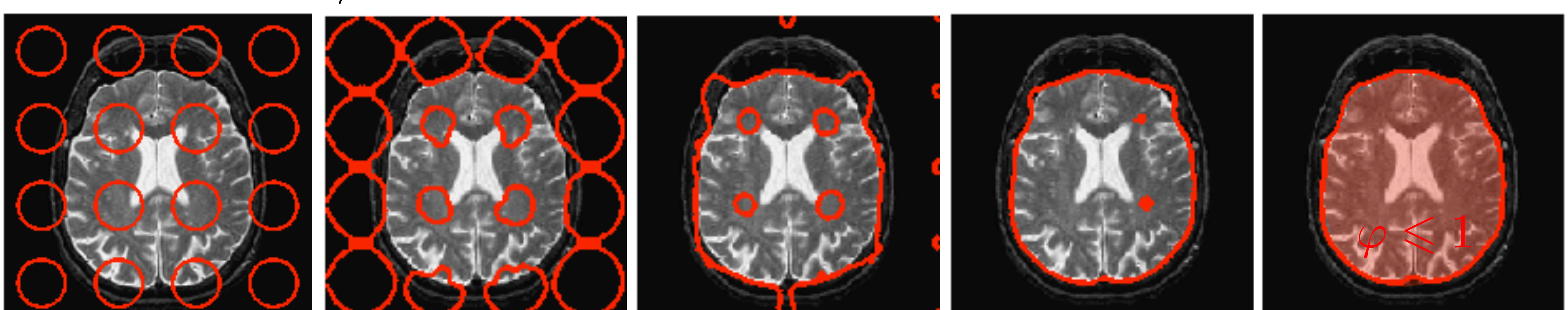
PDE on φ

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \alpha\left(\cdot, \frac{\nabla \varphi}{\|\nabla \varphi\|}, \mathrm{div}\left(\frac{\nabla \varphi}{\|\nabla \varphi\|}\right)\right) \|\nabla \varphi\|$$





Mumford-Shah / Chan-Vese evolution:



THE CHEMICAL BASIS OF MORPHOGENESIS

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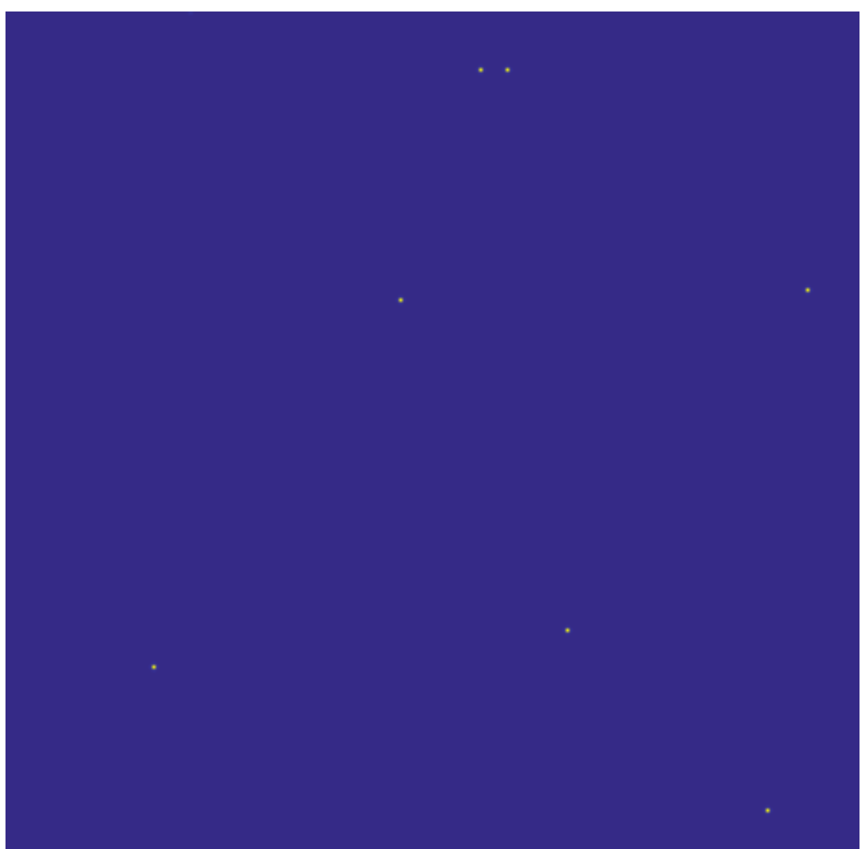
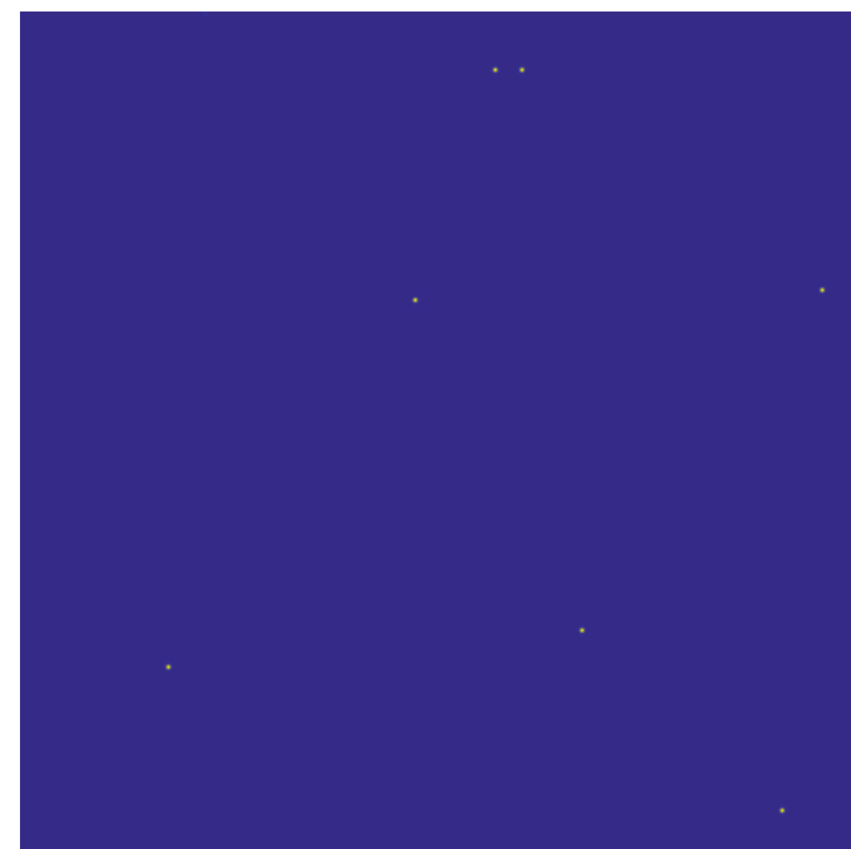
Gray-Scott Model:

$$\frac{\partial u}{\partial t} = D_u \nabla^2 u - uv^2 + F(1-u),$$
$$\frac{\partial v}{\partial t} = D_v \nabla^2 v + uv^2 - (F+k)v.$$



$F = 0.026, k = 0.0.53$

$F = 0.033, k = 0.062$



$F = 0.055, k = 0.063$

L^2 gradient flow: $\min_f E(f) \longrightarrow \frac{\partial f}{\partial t} = -\nabla E(f)$

Heat equation: $\frac{1}{2}\int \|\nabla f(x)\|^2 \mathrm{d}x \longrightarrow \frac{\partial f}{\partial t} = \Delta f$

TV flow: $\int \|\nabla f(x)\| \mathrm{d}x \longrightarrow \frac{\partial f}{\partial t} = \mathrm{div}\left(\frac{\nabla f}{\|\nabla f\|}\right)$

Gradient: $\nabla f = (\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_1}) : \mathbb{R}^2 \to \mathbb{R}$.

Divergence: $\text{div}(v) = \frac{\partial v^1}{\partial x_1} + \frac{\partial v^2}{\partial x_2} : \mathbb{R}^2 \to \mathbb{R}$.

$$\text{L}^2(\mathbb{R}^2 \to \mathbb{R}) \underset{\text{div}}{\overset{\nabla}{\rightleftarrows}} \text{L}^2(\mathbb{R}^2 \to \mathbb{R}^2)$$

Adjointness: $\text{div} = -\nabla^*$

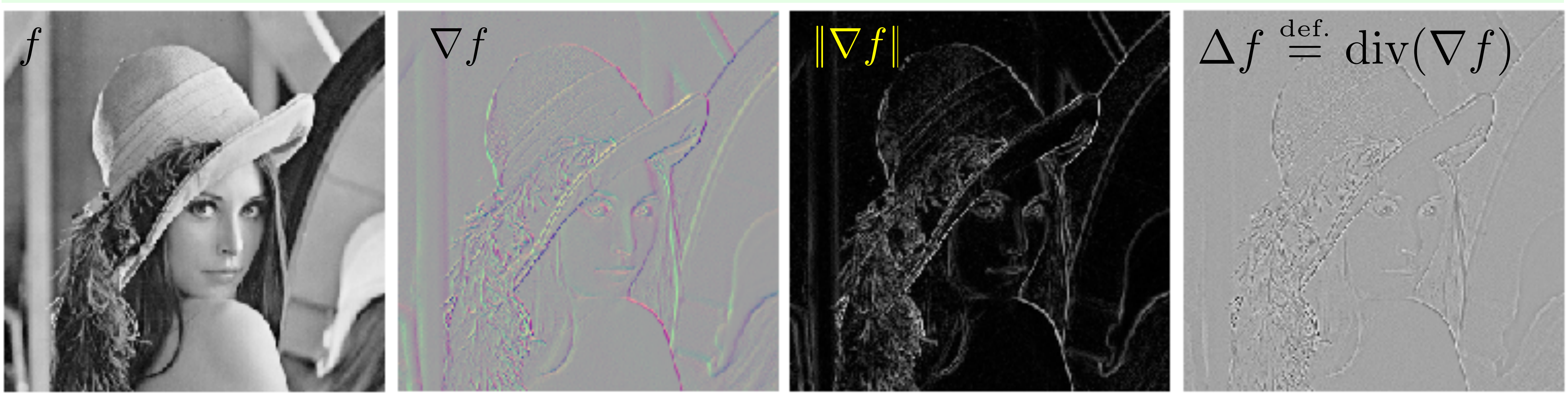
$$\int_{\mathbb{R}^2} \langle \nabla f(x), v(x) \rangle \text{d}x = -\int_{\mathbb{R}^2} f(x) \text{div}(v)(x) \text{d}x$$

$$\nabla f = (f_{k_1+1,k_2} - f_k, f_{k_1,k_2+1} - f_k)_k$$

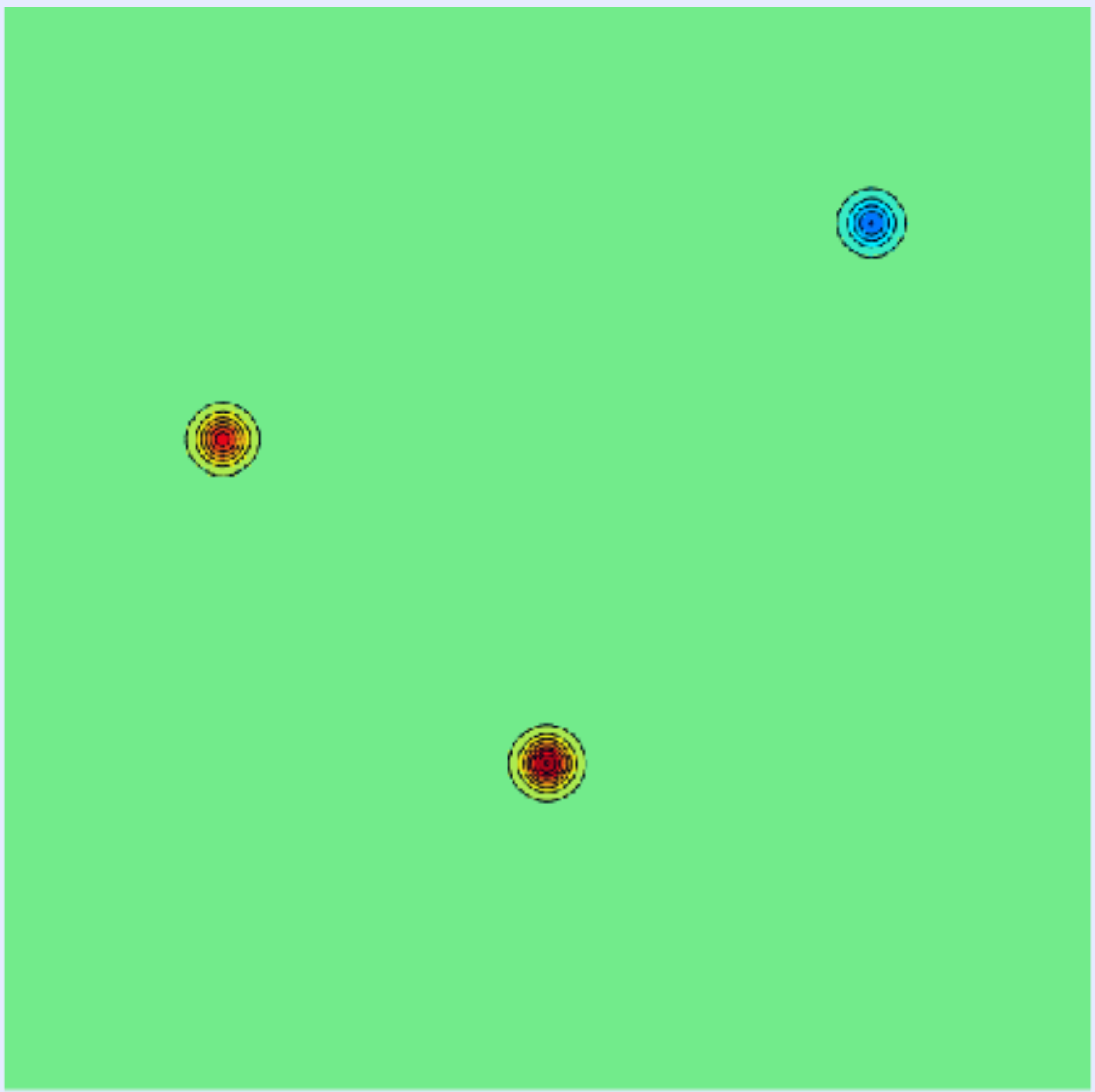
$$\text{div}(v) = v_k^1 - v_{k_1-1,k_2}^1 + v_k^2 - v_{k_1,k_2-1}^2$$

$$\mathbb{R}^{n \times n} \underset{\text{div}}{\overset{\nabla}{\rightleftarrows}} (\mathbb{R}^{n \times n})^2$$

$$\sum_k \langle (\nabla f)_k, v_k \rangle = -\sum_k f_k \text{div}(v)_k$$

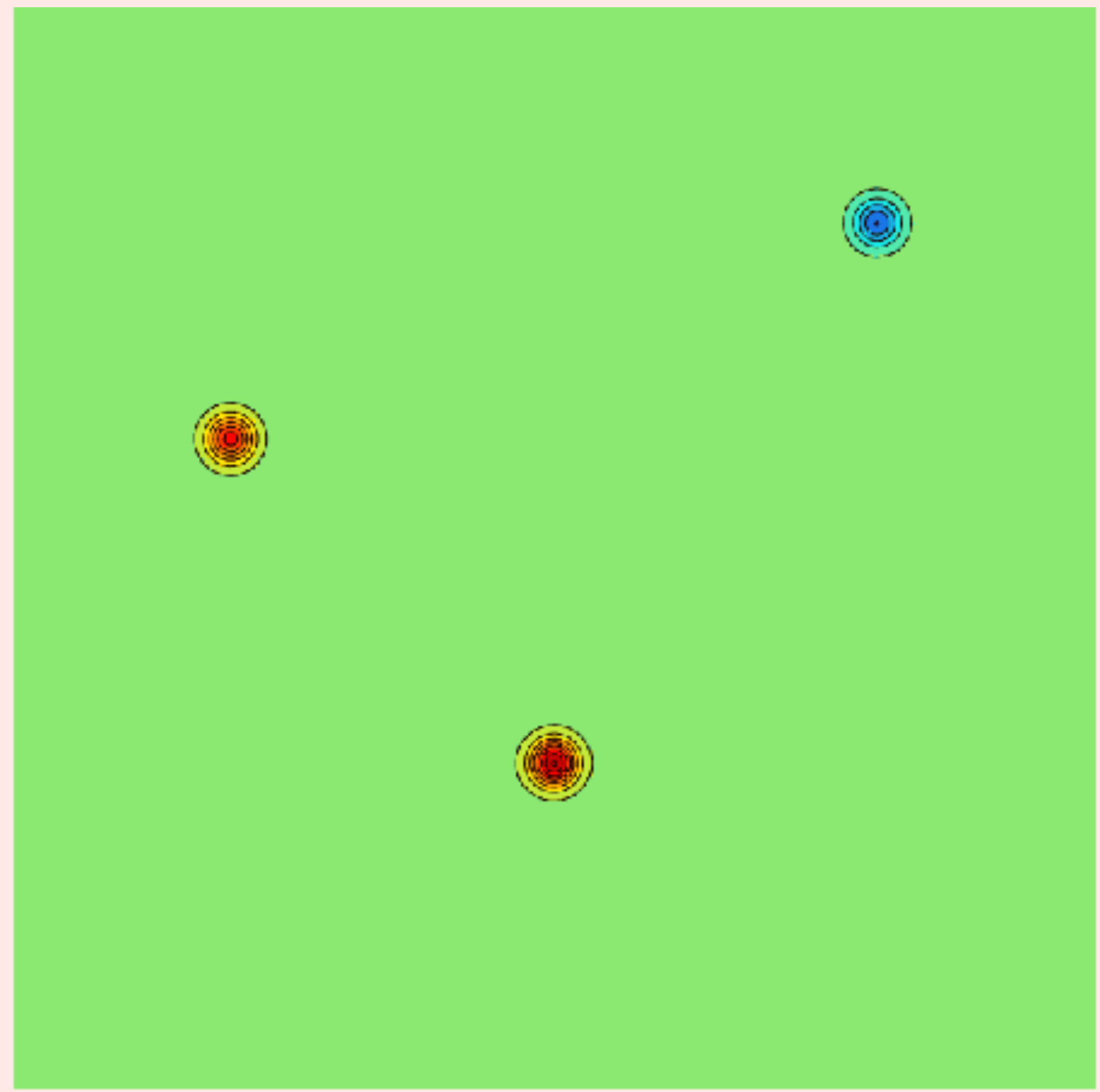


Parabolic: $\frac{\partial f}{\partial t} = \Delta f$



Heat equation

Hyperbolic: $\frac{\partial^2 f}{\partial t^2} = \Delta f$



Wave equation

$$u = {\color{blue}v} + {\color{red}w} \qquad \begin{cases} \operatorname{div}({\color{blue}v}) = 0 \\ \operatorname{curl}({\color{red}w}) = 0 \end{cases}$$